**Proceedings of the 2013 Joint ASME/USCG Workshop on Marine Technology & Standards**
**July 24-25, 2013, Arlington, Virginia, USA**

# Criteria For Eliminating Cyclic Limit For PVHO Flat Disc Windows

**Bart Kemper, P.E.**
Principal Engineer
Kemper Engineering Services, LLC
Baton Rouge, LA

ABSTRACT: Acrylics (Poly Methyl Methalcrylate, or PMMA) are a proven, reliable material for Pressure Vessel for Human Occupancy (PVHO) windows. The current codes and standards reflect a first-generation development of defining a nonmetallic for pressure vessel application. Subsequent investigations in material science as well as decades of operational experience indicates the code-specified 10,000 cycle limit for some flat disc acrylic ASME PVHO windows may be eliminated using criteria consistent with the ASME Boiler and Pressure Vessel Code.

**Acrylics**, or Poly Methyl Methacrylate (PMMA), is a commonly used viewport material for submersible and other Pressure Vessels for Human Occupancy (PVHOs). The initial code rules for acrylic were published over 30 years ago and were highly conservative due to its novelty and lack of long-term data. Industry now has decades of experience as well as various advances in material science. Just as various societies and jurisdictions have updated many of its rules and procedures for conventional pressure vessels, this paper addresses updating the approach to part of safety code pertaining to acrylics in pressure vessel applications.

The American Society of Mechanical Engineers (ASME) is the U.S. code society publishing the Boiler and Pressure Vessel Code (BPVC) used in North America. It has also been adopted in one form or another by other nations. The BPVC has well established rules for using steels and other materials. The use of acrylics is defined in Safety Standard for Pressure Vessels for Human Occupancy, also known as PVHO-1. [1] This code builds on BPVC and adds the application-specific needs for PVHO, to include viewports.

PVHO Section 2-1.2 (d) limits the number of cycles to 10,000 cycles. While it is the result of dozens of papers and hundreds of experiments, this limit is not based strictly on material properties. PMMA has a flexural endurance limit of approximately 2,600 psi. [2] This is shown in Fig. 1. Like most other materials, notches and other stress concentrators reduce fatigue life, as shown in Fig. 2 [2]. It is noted there are several grades and types of PMMA approved for PVHO applications (Table 2.6 [3]). Not all types of PMMA meet the requirements for PVHO applications.

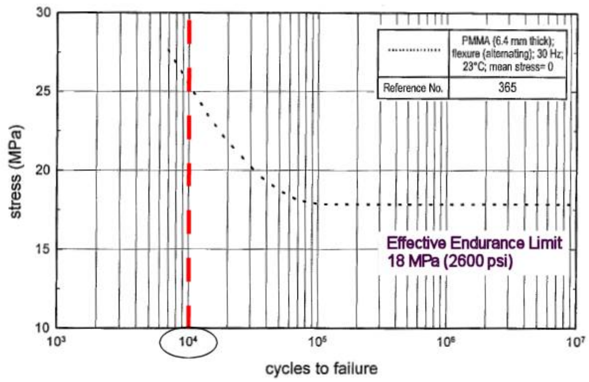


*Fig. 1. Flexural fatigue life, annotated to highlight 10,000 cycles and the endurance limit. [2]*

Based on the Figs. 1 and 2, it appears windows can be designed to be well below the fatigue curve and be at "infinite life." A conservative initial value for a "limiting cyclic stress" (LCS) is 12.55 MPa (1820 psi). This value is 70 percent below the literature values, allowing for variance in PVHO-compliant PMMA blends, minor defects allowable in PVHO-2 (Guidelines for In-Service PVHOs), and provides a design margin with respect to strain energy.

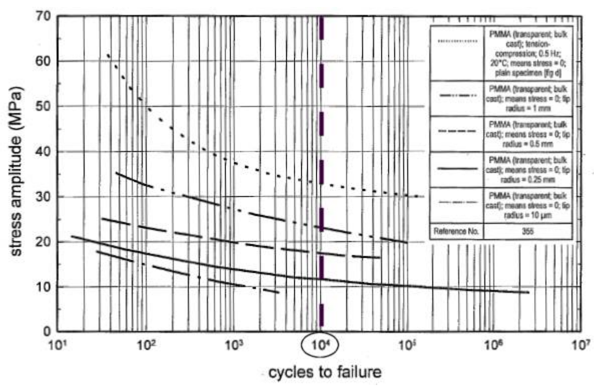


*Fig. 2. Fatigue curves showing the effect of cracks on fatigue life. The 10,000 cycle limit in PVHO is annotated on the figure. [2]*

The 10,000 cycle limit has potential economic impact on submersibles and some hyperbaric applications that can expect many cycles per day. Many windows reaching 10,000 cycles exhibit no flaws or defect and give every indication they are as serviceable as the day they were installed. The costs associated with the replacement, test procedures, and associated downtime can be significant.

However, according to PVHO the 10,000 limit can only be extended by empirical testing. Section 2-2.7.9 [1]. This allows adding one design cycle for every two cycles tested above an initial 10,000 cycles tested. Each cycle requires at least 15 minutes at full pressure (or 1.5 times the time needed to stabilize creep, whichever is greater) followed by a depressurization of 10 minutes (or 1.5 times the time needed to stabilize creep, whichever is greater).

If one assumes an average of two cycles per hour with continuous testing, it would take about 6.8 months to reach the 10,000 cycle threshold before additional cycles could be credited. Only that specific pairing to a specific window mount with that maximum tested temperature can receive this credit. This can create a significant commercial burden and market entry delay for a given design.

## Fatigue Mechanism

Fatigue failure can be described as the progressive and localized structural damage when a material is subject to cyclic loading. PMMA is a viscoelastic material. [4] For PVHO windows, the allowable design temperatures range from 10˚C to 66˚C (50-150˚F) (Table 2-3.4-1 [1]). These temperatures are well below the glass transition temperature of PMMA, which ranges from 110-120˚C (230-248˚F). Unlike traditional pressure vessel materials, it is sensitive to creep in this temperature range with significant deviations in ultimate strength, yield strength and other properties. [5]

The specific fatigue mechanisms for PMMA are still being established. Some research indicates that fatigue competes between two mechanisms: polymer chain slippage and chain scission. It also indicates increased molecular weight due to crosslinking does not uniformly increase fatigue properties. While many previous findings indicate high molecular weight PMMA has superior fatigue properties, tests with a higher mean stress, or R, indicate lower molecular PMMA displays better fatigue properties. This is due to competition between chain slippage and chain scission is related to the alternative stress levels R and the stress intensification factor K are driven by different molecular interactions. [6]

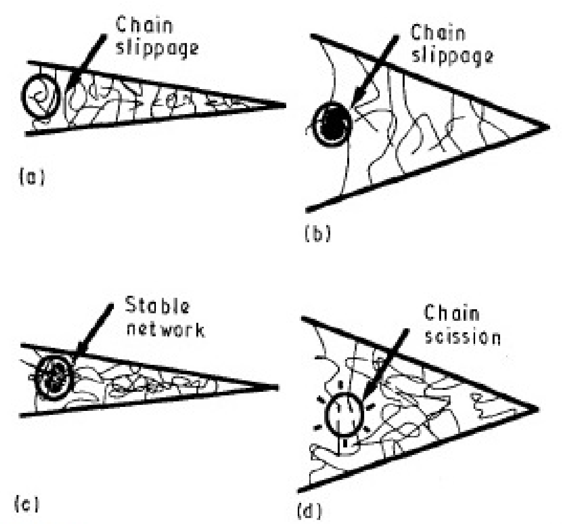


*Fig. 3* Schematic diagram showing the fatigue processes in each $M_w/R$-ratio condition: (a) low-$M_w$/low-$R$, (b) low-$M_w$/low-$K_{max}$ (c) high-$M_w$/low-$R$ and (d) high-$M_w$/high-$K_{max}$ [6]

Investigating crazing deformation during fatigue indicates no cyclic rate dependence or waveform dependence for crack propagation of similar crack geometries as measured by the stress intensification factor K. [7] Subsequent work indicates that flaws previously discounted in much of the previous research in the field as insignificant serve as the nucleus for craze formation and that von Mises stresses indicating yielding can be used to predict plastic conditions needed to create crazing. [8] Other work points more to specific activation energy levels. Investigating fatigue-crack propagation (FCP) indicates the crack propagation is essentially the transition from the “glassy” to the β transition phase as a viscoelastic material. [9] High strain rates and high cyclic rates create significant heat within the structure with the material acting at or above the glassy transition temperature, which then leads to fatigue failure. This same research further indicated no influence of molecular weight distribution on deformation kinetics as well as supports the development of fatigue endurance limits under conditions below the glass transition temperature. [10]

## PVHO Window Design Process

The key element of agreement within material science literature is that PMMA and similar approved PVHO glassy polymers will not exhibit fatigue failure without an existing crack at temperatures consistent with PVHO application. This is supported by no reports of PVHO-compliant window failures through open source methods. It is also consistent with the PVHO window design process results in design margins greater than PVHO shell. This results in the first design failure mode being a structural element other than the window. With designs using polymer seals or gaskets, the polymer structures will fail before the windows or window seat in accordance with the informal “leak before failure” design philosophy cited during open sessions of the PVHO subcommittees.

Other code requirements contribute to protection from PMMA fatigue mechanisms. Windows must be replaced if they have a visible crack or hazing that obscures vision. PVHO applications require cycles that last many minutes, if not hours or days, per cycle with a maximum pressurization rate of 4.5 MPa/min (650 psi/min). The reduces both the strain rate and cyclic rate below the various thresholds reported in the cited literature. [6] [10] [11]

Creep does occur with PMMA within the PVHO application range. However, the stress levels typical for a properly designed PVHO window results in negligible creep in the life of the window. [3] These stress levels are well below the levels associated with damage, as excessive creep is also associated with crazing formation. [12] Maximum code allowable axial displacement of a flat disc window is 0.30 of the bearing gasket uncompressed thickness. This deflection includes creep, window deflection, and pressure-induced gasket compression. [1] This further indicates the window is well within the glassy phase and not heavily stressed, otherwise the deflection would be greater.

## Flat Disc Windows

Flat disc windows are the most common PVHO window due to simplicity and cost. Two designs will be compared. The first is known to safely exceed the 10,000 cycle limit. Atlantis Submarines reports having a PVHO flat disk window design that is under continuous testing and is now over 200,000 cycles. This allows 100,000 additional cycles for a total of 110,000 cycles and counting. The window is 647 mm (25.5 in.) in diameter with a 82.6 mm (3.25 in.) thickness with a maximum service temperature of 38 ˚C (100 ˚F) at 0.46 MPa (67 psi). This design provides a concrete example of a design with extended cyclic life.

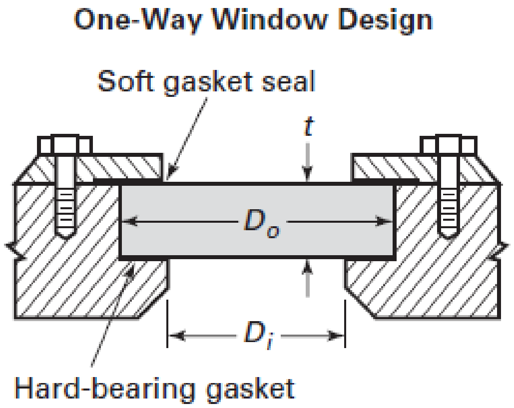


*Fig. 4. One of the standard configurations for a flat disc PVHO window [1] and is consistent with the submarine window design being evaluted. The cyclic tests to failure did not have a retaining ring nor appropriate hard bearing gasket. [13]*

The second design was to be used for the US Navy's Experimental Diving Unit (EDU). The 112.5 mm (4.43 in.) diameter, 25.9 mm (1.02 in.) thick window was rated for 3.1 MPa (450 psi) at 49 ˚C (120 ˚F). This window was cyclically loaded to failure at pressures ranging from 24.1-37.9 MPa (3500-5500 psi). [13] The lowest test pressure was 7.7 times the design pressure. This design provides a concrete example of cyclic failure. It failed after 120 cycles. The two geometries will be examined using FEA. In both cases the acrylic is MIL-P-8184, which has detailed published properties. [5]

It should be noted the cyclic loading-to-failure test was conducted without a retaining ring to allow for unrestrained rotation of the window about its contact with the inner edge of the window seat. This is not a PVHO-complaint design. The gaskets specified in PVHO would most likely fail during these sizable overpressures. As the higher pressure side was loaded, the low pressure side was forced into an increasing cuplike shape so only the inner edge of the seat was in contact with the window. This increased the contact pressure on the 0.025" (0.67 mm) neoprene-impregnated cloth gasket to a knife edge with a linear pressure of 5,625 lbf/linear inches at the lowest pressure of 3500 psi. The gasket could not contribute to the structural response and is omitted in the FEA.

To further compare the impact of geometry, the submarine window was modeled without its gaskets at 7.7 times its maximum pressure, similar to the EDU window. The EDU window model was modified to comply with PVHO code by having a hard bearing gasket plus a soft gasket seal restrained on the upper portion consistent with a retaining ring compressing the gasket.

**Computational Models**

Section VIII, Division 2, Part 5 (Design By Analysis) allows for the use of FEA for the design of pressure vessels and its components. Implicit nonlinear (elastic-plastic) analysis as well as linear (elastic) analysis are the two basic types of modeling cited in Section VIII, Div. 2. While explicit nonlinear analysis provides a more precise and detailed examination of failures, the intent is to ensure the design is well within safety limits for the given operation. Constraining the computational methods to those already approved for ASME pressure vessel design maintains a consistent set of tools for the industry.

FEA has been used successfully to model PVHO windows and was instrumental in developing the current design criteria. [3] For flanged hemispherical windows it was concluded stresses were linear to at least 50 percent of collapse pressure for all hemispherical shells. [14] For spherical hulls there was excellent agreement between the experimental results and the FEA results. [15] However, the art of the time does not indicate the use of contact elements nor modeling the interaction of the gaskets, o-rings (as applicable), and retaining ring with flat disc window applications.

Fig. 5 is an example of a flat disc window that failed during pressurization 12 times its rated pressure [13]. The nonlinear FEA stresses for this test are consistent with the observed failure mode. This failure is consistent with the examples in the study as well as other studies of flat discs subjected to similar over-pressurization. [16]. Based on study of the text fixture, the sharp edge of the bearing surface's inner diameter creates highly localized peak stress in the window as well as is the location of the greatest strain energy density. This makes this the most likely location for crack initiation.

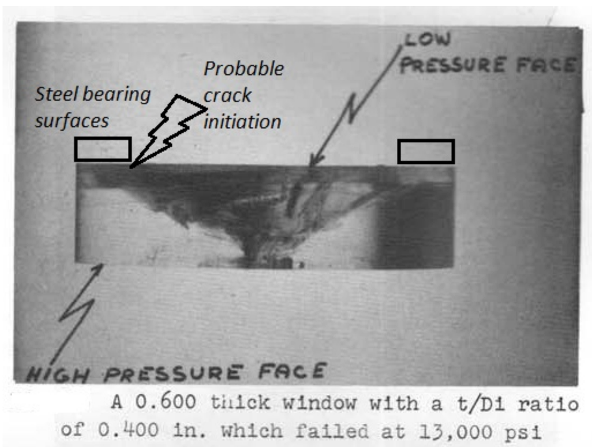


*Fig. 5a. A flat disc failure due to excessive pressure using the same test apparatus in the cited cyclic testing. [16] Annotation for the steel bearing surface symbol and probable crack initiation point is added by the author.*

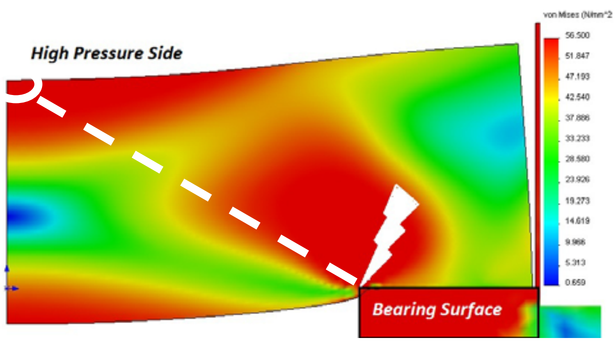


*Fig. 5b. VonMises stress of the 37.9 MPa (5,500 psi) test. The model is 2D axisymmetric with the axis of symmetry being the window centerline, which is the left side of the plot. The stress plot range is limited to 56.5 MPa (8200 psi), the tensile strength at test temperature. This sample failed in the first cycle. [13] The peak strain energy as well as peak window stress is at the corner-to-window interface indicated by the bolt.. The largest area of above-tensile PMMA is the center of the higher pressure face (circled region on the left.). This is consistent with the inner edge of the mount mechanically inducing a crack on the low pressure side, then propagating up and to the center of the window, creating a conical failure plane consistent with the photograph in Fig. 5a. as Type II crack propagation. [17]*

The window assembly geometry favors an axisymmetric approach. A 3D analysis would be valid but would not be needed to only evaluate nonlinear stresses for fatigue life. Contact elements must be used to allow the window to move relative to the window seats and to allow the gaskets to deform. The steel and elastomer items can be modeled as linear materials. The nonlinear stress-strain curves for MIL-P-8184 PMMA specification were developed using information developed by the US Air Force and is publically available. [5]

Contact elements must be used to allow each component to displace relative to each other. Friction can be neglected as a conservative assumption. The analyst should ensure the proper material properties are used for the elastomer elements and not use a generic "rubber" value.

Nonlinear FEA typically uses "pseudo time" as a method to apply loads incrementally or in a specific pattern. Actual time can also be used, given the loading and unloading rate for pressure is limited in PVHO-1. Regardless of using pseudo time or actual time, the analyst must apply the retaining ring force on the soft gasket or o-ring prior to applying the pressure. Failing to do this will result in inaccurate results. It will also allow the analyst to assess the mechanical response of the seals, gasket, and window due to the window assembly

For the submarine window, a force was applied such that the retaining ring was flush against the window mount. Force in excess of what is required to close the gap is transferred into the steel structure. The stresses in the steel structure are ignored for the purpose of evaluating the window stresses.

For the EDU window, there was no retaining ring as part of the test apparatus. The stated intention of test was to induce failure using assumptions and conditions that would be more conducive to failure than the actual installation. [13]. Rather than redesigning the test apparatus into a fully PVHO compliant viewport assembly, the "at service pressure" model had the hard bearing and soft gasket added to the model then the soft gasket was displaced 30 percent in compression.

FEA can also be used to examine creep. Many FEA codes incorporate some form of the Power Law to incorporate creep behavior into nonlinear materials. However, the ASME PVHO-1 code limits the impact of creep through the limitations and specifications for design details, loads, and service conditions.

**FEA Results**

The two window assemblies are for different pressures and temperatures. The proposed evaluation criteria was applied such that the "at rated pressure" analyses used the proposed Limiting Cyclic Stress value. The maximum stress value was truncated to the LCS such that all values that are red are over, at, or very close to the screening criteria.

The unrestrained, high pressure analyses were to examine known or likely failure modes. The stress plots were truncated at the maximum tensile strength for the PMMA at that temperature. Von Mises stress plots are in Fig. 6 a-d.

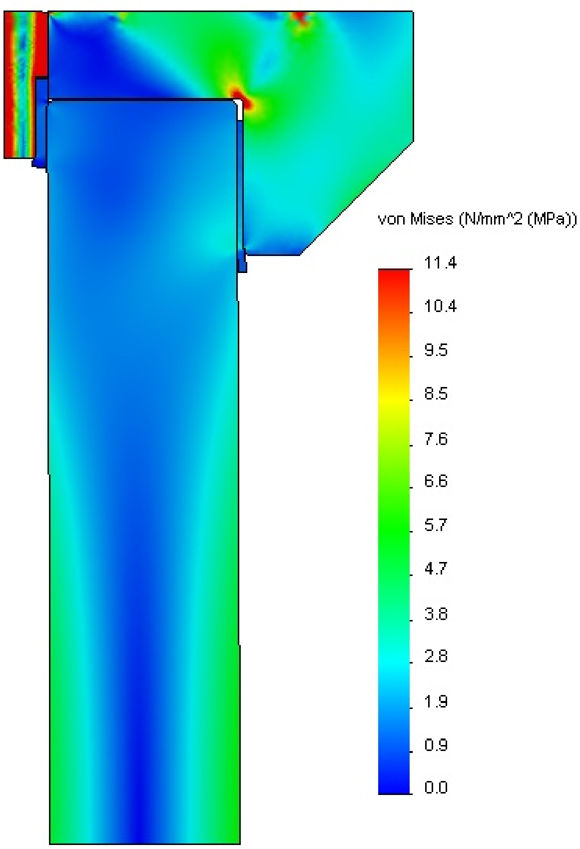


*Fig. 6a. A flat disc submarine window exhibiting infinite life characteristics. It has been physically tested to 200,000 cycles and beyond, well in excess of the 10,000 cycle limit used in PVHO-1. The temperature is 38˚C (100˚F) and the pressure is 0.46 MPa (67 psi). The LCS is 11.35 MPa (16.57 psi)). The von Mises stress plot indicates the peak value in the window are approximately half of the LCS. This window seat assembly meets the computational criteria for eliminating the 10,000 cycle limit. In addition, there is no significant differences is stress patterns or stress magnitude between the beginning or end of the pressure cycle.*

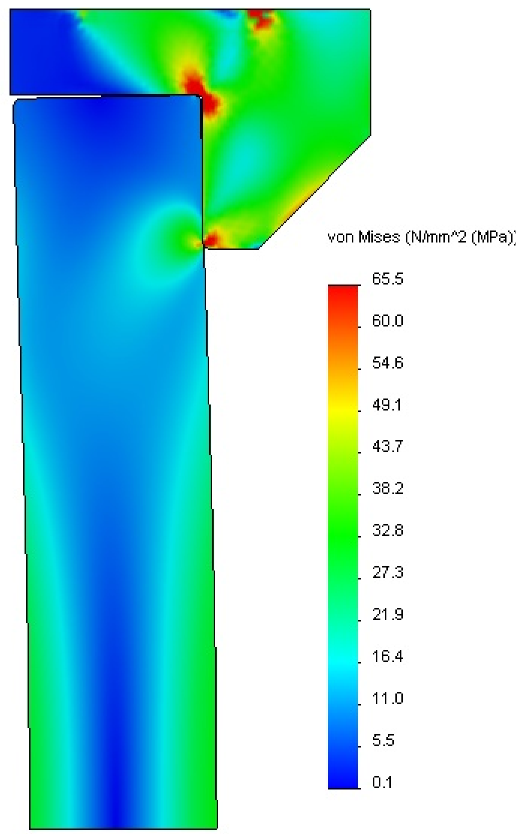


*Fig. 6b. The submarine flat disc window has had the elastomer structures removed and retaining ring removed. It is free to rotate due to pressure induced deflection. The tensile strength for the window is 65.5 MPa (9,500 psi). The inner diameter of the window mount creates a significant stress concentration in the window. This location is consistent with the failures cited in similarly constructed tests[13] [16]. While peak stress in the window is about 50 MPa (7251 psi) and is below the ultimate strength, it is well into the nonlinear portion of the stress-strain curve. There are significant residual stresses in the location when the pressure cycle is completed.*

*This does not indicate an immediate failure but elevated residual stresses once all constraints are released is part of the mechanism associated with cyclic failure. Residual stresses are not a guarantee of a cyclic failure. The stress pattern at the bearing surface illustrates the role of the bearing gasket distributing the loads and resultant stresses.*

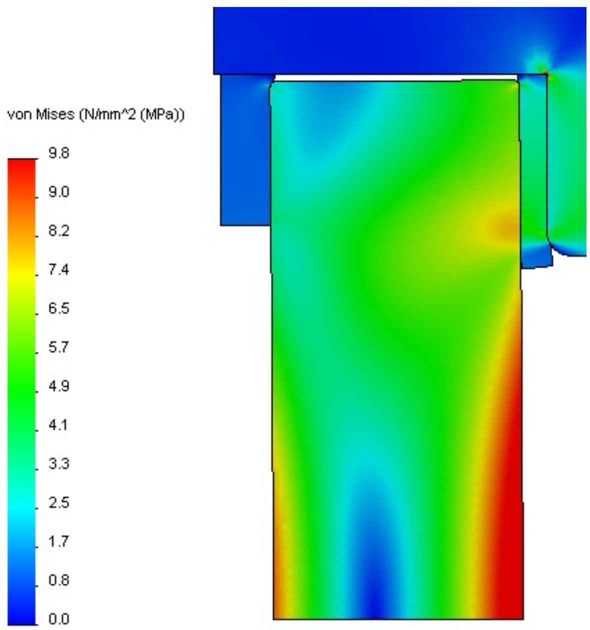


*Fig. 6c. The EDU chamber model modified by the addition of gaskets and analyzed at its operating pressure 3.1 MPa (450 psi). Peak stress is 12.2 MPa (1766 psi). While this is below the LCS at ambient conditions, the 49˚C (120˚F) temperature reduces the LCS proportionally with the change in tensile strength, reducing the LCS to 9.8 MPa. (1421 psi). Stresses are far below the tensile strength of 56.6 MPa (8200 psi) and there is no significant difference between the stress levels or patterns at the start and end of the pressure cycle. However, since the peak stresses in the window did not meet the proposed LCS value, this window would not be recommended to have its 10,000 cycle limit removed without physical testing per ASME PVHO-1.*

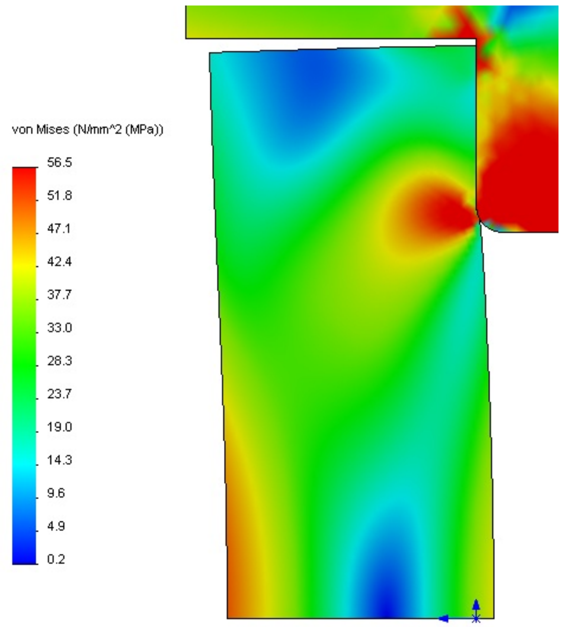


*Fig. 6d The EDU chamber model as tested at 24.1 MPa (3,500 psi). This is 7.7 times its rated operating pressure. The von Mises stress range is truncated at the tensile strength in order to highlight those areas above 56.6 MPa (tensile strength). Peak stress is 58.0 MPa, which is above tensile. There are significant residual stresses at the end of the pressure cycle. The amount of red is not as dramatic as Fig. 5, which is the same model at 37.9 MPa (5,500 psi), it is significant. This test experienced failure at 120 cycles. In comparing the results of the five tests of this model, the results appear consistent with a crack forming at the inside diameter of the bearing surface at the large and sharply defined stress concentration occur, then having the crack propagate over a short number of cycles (less than 150) until catastrophic failure. These failures are cyclic in nature but are not appear to be due to a classic pure fatigue mechanism due to the short number of cycles. It is more consistent with mechanical crack formation (Type II) coupled with fatigue crack propagation, consistent with the curves in Fig. 2.*

TBL 1 FEA RESULTS

| **Model and Conditions** | **Peak Stress** | **LCS** | **Tensile** | **Comments** |
|---|---|---|---|---|
| Submarine @ operating pressure | 6.6 MPa | 11.35 MPa | 65.6 MPa | No fatigue issue |
| Submarine @ 7.7 x opn. pressure | 50.0 MPa | 11.35 MPa | 65.6 MPa | Likely failure |
| EDU @ operating pressure | 12.2 MPa | 9.8 MPa | 56.6 MPa | Requires testing |
| EDU @ 7.7 x opn. pressure | 58.0 MPa | 9.8 MPa | 56.6 MPa | Failure |

**Proposed Criteria for Eliminated Cyclic Limit**

The original safety and engineering codes for using transparent polymers as pressure vessel materials relied heavily on empirical data. [3]. PMMA is seeing increasing range and depth of applications, to include hyperbaric, biomedical and manufacturing applications. It appears to the author there will be continued research. These proposed guidelines are meant to be as conservative as the original code. They reflect advances in understanding PMMA, no reports of failure of properly maintained flat disc windows, and an extensive body of knowledge of using computational models for PMMA.

PMMA and other glassy polymers are not as well understood as conventional metallic alloys. The intermolecular role of the polymer chains and polymer additives has a significant impact on yield behavior and strain hardening [18]. The glass transition temperature is a critical material change, yet the mechanisms for what this is and how it occurs is still being explored with multiple numeric models providing some of the insights. [4], [9], [19]. Complex constitutive models for glassy polymers are developed, refined, and replaced. [4], [20], [21]. However, as previously discussed, the temperature range associated with human safety as well as the stated allowable temperatures in the ASME PVHO-1 and PVHO-2 safety codes limits the service application of PMMA to conditions that are well understood and proven to be predictable.

Based on this, the cyclic loading requirement can be eliminated if specific criteria are met. Those criteria are as follows:

1. The window design and window material comply with PVHO-1 standards and has supporting documents.
2. The window has been maintained according to PVHO-2 (In-Service Guidelines for PVHOs) and has supporting documents. This document specifies under what conditions a window must be removed from service. If these conditions manifest regardless of cyclic life, the window will be removed.
3. The window has maintained at a temperature range between 50F and the maximum design temperature. This is substantiated with the supporting documents.
4. The window is used in temperature stable environment that is shielded from UV rays, such as a hospital medical chamber or submarine windows in tropical or subtropical waters that are kept submerged during all modes of operation.
5. The windows which are documented to not have been exposed to X-rays or similar hard radiation, which is also known to degrade PMMA performance.
6. The window has exceeded 8,000 cycles without any cracks or other flaws developing. This number of cycles provides sufficient operational time to allow any flaws, cuts, or cracks to propagate and be detected while still providing the owner with 20 percent of the cyclic life to perform the work needed to waive the cyclic limit if all conditions are met.
7. The window has is still in its original installation fixture. This is due to creep issues. A window will develop over time a specific geometry form-fitted to its fixture. Placing a creep-deformed window in a new fixture could introduce sufficient geometric differences to invalidate eliminating the cyclic limit.
8. A nonlinear stresses analysis is performed incorporating the seat, retaining ring, o-rings (if applicable), gasket, and window. Contact elements are used to separate the structural elements. The true stress-strain curve for the maximum design temperature is applied to the acrylic window. The remainder of the structural items may be modeled with linear materials. The peak stress in the window shall be 12.55 MPa (1820 psi) or less at 20˚C (68˚F) unless there is a fatigue curve for the specific PMMA blend used, in which case the peak stress in the acrylic window should be 80% of that curve's endurance limit. This threshold is termed the "limiting cyclic stress", or LCS.
9. The fatigue curve in Fig. 1 may be used for values less than infinite life providing the stress values are reduced to 70 percent of the plotted value. A fatigue curve for the specific PMMA takes precedence over the

curve in Fig. 1. The stress values of the PMMA-specific curve will be reduced to 80% of the plotted stress values instead of 70% due to the greater specificity of the material analyzed.

10. The LCS will be reduced proportionally to the decrease in tensile strength at maximum operating temperature compared to 20 ˚C (68 ˚F). If temperature-specific fatigue curves for the specific PMMA blend is available, those values will be used and 80 percent of the endurance limit will be used. The LCS will not be increased based on temperatures lower than 20˚C (68˚F).
11. There should be no significant stress change when comparing the beginning of the pressure cycle to the end of pressure cycle, in which both cases pressure = zero. The stresses due to the clamping force of the retaining ring should remain consistent when comparing the beginning and end of the pressure cycle.
12. A competent professional engineer, registered in one or more of the U.S. states or provinces of Canada, or the equivalent in other countries, and experienced in the design of PVHOs, shall certify that the PVHO window (including the full viewport assembly) was appropriately designed and maintained per ASME PVHO-1 and PVHO-2, appropriate FEA was performed, and all other criteria was met such that the specific window design in the specific installation site may operate beyond the 10,000 cycle limit as long as continues to otherwise comply with ASME PVHO-2.

For new designs, all elements of the above must be met except the number of cycles reached. The engineer's report certifies all aspects listed above other than the 8000 cycle limit. Providing the window is maintained to PVHO-2 in its original fixture and no cracks or defects develop, the window may be kept in service when it exceeds 10,000 cycles.

Windows that do not meet the cyclic limiting stress criteria can still be qualified experimentally per PVHO-1. Useful remaining life can be estimated using methods in ASME Fitness-For-Service (API 579-1/ASME FFS-1) providing the fatigue curve is per Item 9. This paper does not address the 40,000 hour limit for PVHO windows.

Further work needs to done to examine the other types of PVHO windows as well as the 40,000 service life limit. Detailed research and testing regarding specific PMMA compositions will form the basis for more precise material modeling and performance prediction.

Special Thanks:
The author gratefully recognizes the information provided by Atlantis Submarines regarding their testing procedures and by Blanson, Ltd. regarding acrylics manufacturing and PVHO-grade acrylics.

About the author:
Bart Kemper, P.E. a 1992 graduate of Louisiana State University. Kemper is the principal engineer for Kemper Engineering Services, LLC. He serves on the ASME PVHO Codes and Standard Committee and its various subcommittees and working groups. He has authored numerous patents and peer reviewed papers and is a member of ASME, NSPE, SNAME, ASM, and other technical societies. He is also an US Army Corps of Engineers officer as a member of the Army Reserves.

References

1. Safety Standard for Pressure Vessels for Human Occupancy (ASME PVHO-1-2012) American Society of Mechanical Engineers, NY. 2012.
2. McKeen, Laurence W. Fatigue and Tribological Properties and Plastics and Elastomers, Plastics Design Library, Norwich NY, 1995.
3. Stachiw, Jerry D Handbook of Acrylics. . Best Publishing Company, Flagstaff AZ, 2003.
4. Lakes, Roderic. Viscoelastic Materials. Cambridge University Press, NY NY, 2009.
5. Hassard, Richard S. MIL-HNDK-17A, Plastics For Aerospace Vehicles. Part II Transparent Glazing Materials. US Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base. Jan. 1973.
6. Clark, T.R., Hertzberg, R.W., Mohammadi, N. "Fatigue mechanism in poly (methyl

methacrylate) at threshold: effects of molecular weight and mean stress." Journal of Material Science, Vol. 28 (1993) p. 5161-5168.

7. Imai, Y, Ward, I.M. "A study of craze formation in the fatigue fracture of polymethlmethacrylate." Journal of Materials Science 20 (1985) 3842 – 3852
8. Bucknall, C.B. "New Criterion For Craze Initiation." Polymer. Vol. 48 (2007) p. 1030-1041.
9. Cheng, W.M, et al. "Mechanical behavior of poly(methyl methacrylate), Part 3, Activation process for fracture mechanism." Journal of Materials Science 25 (1990) p. 1931-1938.
10. Janssen, R.P.M., et al. "Fatigue Life Predictions for Glassy Polymers: A Constitutive Approach. Macromolecules Vol. 41 (2008), p. 2520-2530.
11. Fleck, N.A., Stronge, W.J., Liu, J.J. "High Strain-Rate Shear Response of Polycarbonate and Polymethyl Methacrylate." Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences, Vol. 429, No. 1877 (June 8, 1990), p.459-479.
12. Lua, W. et al. "Creep behavior of poly(methyl methacrylate) with growing damage." Materials Science and Engineering A: Vol 483-484 (2008) p. 580-582
13. Stachiw, J.D. "Technical Note N-1127: Flat Disc Acrylic Plastic Windows For Man-Rated Hyperbaric Chambers at the USN Experimental Diving Unit." Naval Civil Engineering Laboratory, Port Hueneme, CA. Nov. 1970.
14. Stachiw, J.D., Maison, J.R. "Flanged Acrylic Plastic Hemispherical Shells for Undersea Application. ASME Journal of Engineering for Industry. Publication 73-OCT-A. October, 1973.
15. Snoey, M.R., Katona, M.G. "Stress Analysis of a Spherical Acrylic Pressure Hull. ASME Journal of Engineering For Industry. Paper No. 70-UnT-B. Aug, 1970.
16. Stachiw, J.D. "Critical Pressure of Flat Acrylic Windows Under Short-Term Hydrostatic Loading." ASME Publication 67-WA/UNT-1. March, 1967.
17. Archer, J.S., Lesser, A.J. "Shear Band Formation and Mode II Fracture of Polymeric Glasses." Journal of Polymer Science: Part B:Polymer Physics. Vol. 49 (2010), p. 103-114.
18. Hoy, R.S., Robbins, M.O. "Strain hardening in bidisperse polymer glasses: Separating the roles of chain orientation and interchain entanglement." The Journal of Chemical Physics. Vol, 131, 244901, (2009)
19. Roth, C.B. et al. "Molecular Weight dependence on the Glass Transition Temperature in Free Standing PMMA films." European Physics Journal E. Vol. 20, p. 441-448 (2006)
20. Alcoutlabi, M., Martinez-Vega, J.J. "Modeling of the viscoelastic behavior of amorphous polymers by the differential and integration fractional method: the relaxation spectrum H(τ)". Polymer 44 (2003). P. 7199-7208.
21. Lee, E-W., et. Al. "Viscoelastic and Non-linear Mechanical Behavior of Lightly Crosslinked PMMA." Proceedings of the Society for Experimental Mechanics. 1-4 June, 2009, Albuquerque, N.M.